Selection Rules Imposed by Charge Conjugation.

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Summary. — A systematic study is made of the invariance of field theories under charge conjugation in order to indicate *all* possible selection rules due to charge conjugation alone or combined with charge symmetry or charge independence. It is shown that when isotopic spin formalism is used, invariance under charge conjugation corresponds to conservation of isotopic parity.

As is well known, present theories are invariant with respect to charge conjugation ([1,2]), i.e. there is a complete symmetry between the two « charge conjugate » states of particles whose fields are described by non-hermitian operators (complex wave function in the usual loose terminology), and the exchange of these two charge conjugate states is called charge conjugation. Fields described by hermitian operators (real wave functions), as the electromagnetic field, contain particles with only one state (no antiparticles); these self-charge conjugate particles will be called « strictly neutral ». The theory of « strictly neutral » particles of spin 1/2 is due to MAJORANA. ([2])

It is expected that invariance of the theory under charge conjugation gives selection rules: some of them have been found by FURRY ([3,4]). In the same

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([1]) H. A. KRAMERS: *Proc. Kon. Ac. Wetensch. Amsterdam*, **40**, 814 (1937).

([2]) E. MAJORANA: *Nuovo Cimento*, **14**, 171 (1937).

([3]) W. H. FURRY: *Phys. Rev.*, **51**, 125 (1937).

([4]) After this work was completed, the paper of L. WOLFENSTEIN and D. G. RAVENHALL: *Phys. Rev.*, **88**, 279 (1952) came to my notice. It contains many of the results of section **1** and **2** and therefore gives a fairly complete list of the selection rules due way, the combination of invariance with respect to charge conjugation and charge symmetry (exchange of protons and neutrons) leads to new selection rules. Those analogous to Furry's theorem have been found, some of them by FUKUDA and MIYAMOTO (5) (see also NISHIJAMA (6), VAN WYCK (6)), the others by the author (7). They have recently been proved quite generally by PAIS and JOST (8). These authors use S-matrix formalism without the use of a power series expansion.

It is intended in this paper to study systematically and to indicate all other possible selection rules due to charge conjugation (alone or in combination with other invariances). Indeed selection rules due to charge conjugation are of fundamental character; they are due to the invariance of the formalism with respect to given groups. Our program is therefore to determine these groups, it then will be an easy matter to indicates *all* possible selection rules. This deduction does not make any assumption about the possible methods for solving the starting equations.

For instance: charge independance of nuclear forces is obtained by the assumption of conservation of isotopic spin (invariance with respect to the group R_3 of rotations in the three dimensional isotopic space). We shall see that charge conjugation is linked with the conservation of isotopic parity (invariance under reflexions in isotopic space). For instance, to include charge conjugation considerations in the study of π-mesons (assumed to be pseudoscalar symmetric) one shall consider them as *polar* vectors in isotopic space.

Aside electrodynamics, for the sake of simplicity, only the usual meson theories (9) will be studied in this paper; the extension of the method to other field theories will be obvious.

In the application of new selection rules, it is useful to remember the other selection rules. Those due to energy momentum conservation are evident; the complete list of those due to angular momentum and parity conservation does not seem generally known and is given in appendix.

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to charge conjugation alone. Since our method is more general (we do not need explicit calculations) and complete, it seems desirable to present a self-contained treatment. The results already obtained by WOLFENSTEIN and RAVENHALL will be denoted by the reference (4).

(5) H. FUKUDA and Y. MIYAMOTO: *Progr. Theor. Phys.*, **4**, 389 (1949).

(6) C. B. VAN WYCK: *Phys. Rev.*, **80**, 487 (1950); K. NISHJIMA: *Prog. Theor. Phys.*, **6**, 614 (1951).

(7) L. MICHEL: Chapter 3 of *Progress in Cosmic Ray Physics*, esp. p. 142 to 144 (Amsterdam, 1952).

(8) A. PAIS and R. JOST: *Phys. Rev.*, **87**, 871, (1952). I wish to thank Drs. PAIS and JOST for communication of their manuscript prior to publication.

(9) N. KEMMER: *Proc. Roy. Soc. London*, A **166**, 127 (1938).

1. – Electrodynamics.

On the advice of some physicists, group theory will not be (explicitly!) used in this section. Of course, section **1** is only an example of application of section **2**.

1·1. – In what cases will selection rules due to charge conjugation appear?

To charge conjugation corresponds an operator C which exchanges positons and negatons (the two states of charge of electron) and transforms photons into themselves. It therefore satisfies ([10]):

$$C^2 = 1 \; . \tag{1}$$

It is not necessary here to enter into details about the well known question of invariance of electrodynamics under charge conjugation. The operator C commutes with the total Hamiltonian and its eigenvalues $c = \pm 1$ are therefore constants of motion. The eigenstates of C are the states of total charge zero i.e. containing photons and/or ([11]) the same number of positons and negatons. Any state can be decomposed into a sum of two eigenstates of C:

$$C\Psi = \alpha\Psi_+ + \beta\Psi_- \; , \tag{2}$$

with

$$|\alpha|^2 + |\beta|^2 = 1 \qquad \text{and} \qquad C\Psi_\pm = \pm\, \Psi_\pm \; . \tag{2'}$$

The commutation of C with the total Hamiltonian implies that $|\alpha|^2$ and $|\beta|^2$ are constants independent of time.

Let us consider a state Ψ of well defined charge and its charge conjugate $\Psi' = C\Psi$. If the total charge is $\neq 0$, Ψ and Ψ' are two linearly independent state vectors and from $C(\Psi \pm \Psi') = \pm\,(\Psi \pm \Psi')$ we easily deduce that for Ψ (or Ψ'), $|\alpha|^2 = |\beta|^2 = 1/2$. Since this is true for any state of total charge $\neq 0$, no selection rules due to charge conjugation will occur for such states.

We are therefore left with the task of studying the eigenstates of C (since

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([10]) The equation (1) must be read: C^2 is the physical identity. Indeed normalized state vectors are defined only up to a phase factor and C^2 can be equal to a phase factor. There is here no lack of generality to take $C^2 = 1$: see note ([21]).

([11]) The expression and/or is used several times in this paper. « A and/or B » means « either A and B, or A, or B ».

any state of zero charge is a statistical mixture of two of them). One [12] is the vacuum Ψ_0. It is sufficient to compare the eigenvalue of eigenstates of C with that of the vacuum; but, for simplicity, we define the eigenvalue of the vacuum as $+1$.

1·2. *States containing only photons.* – A state with one photon is of the form $A^\mu \Psi_0$ where A^μ is the electromagnetic field. Now, from the commutation of C with the total Hamiltonian one has

$$CA^\mu j_\mu C^{-1} = A^\mu j_\mu \,. \tag{3}$$

The exchange of positons and negatons changes the sign of the current

$$Cj_\mu C^{-1} = -j_\mu \,, \tag{4}$$

therefore

$$CA^\mu C^{-1} = -A^\mu \,, \tag{4'}$$

and

$$CA^\mu \Psi_0 = CA^\mu C^{-1}\cdot C\Psi_0 = -A^\mu \Psi_0 \,. \tag{4''}$$

An easy generalization of this gives:

The eigenvalue of C for states containing n photons is $c = (-1)^n$. This is equivalent to Furry's theorem and applies as well to virtual photons if one uses perturbation theory.

1·3. *Positronium.* – Positronium is the simplest and most interesting case of states with the same number of positons and negatons. We consider both bound and unbound states of positronium.

Indeed C commutes with the energy, momentum, orbital angular momentum, spin, parity operators (they are invariant under charge conjugation). A linear operator that commutes with each of a complete set of commuting

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[12] It is not trivial that the vacuum is an eigenstate of C because it can be described as a degenerate state, as for instance in the indefinite metric formalism (S. GUPTA: *Proc. Phys. Soc.*, A **63**, 681 (1950); K. BLEULER: *Helv. Phys. Acta*, **23**, 567 (1950)) there all states of the vacuum are defined by « no electrons, no transversal photons, n longitudinal photons and n scalar photons present ». As will be seen from (4'') all these states which differ from each other by an even number of photons belong to the same eigenvalue of C.

dynamical variables is a function of them ([13]). Such a complete set commuting with C is easy to find in the case of positronium. Since C commutes with Lorentz transformations we choose as reference of coordinates the center of mass system; then $\boldsymbol{L}^2$, L_z $\boldsymbol{S}^2$, S_z (orbital and spin momenta) for the large components of the electron field form a complete set of commuting dynamical variables. The Pauli principle requires that the states of positronium must be antisymmetrical with respect to space, spin and charge coordinates of the two particles.

The symmetry character corresponding to the exchange of

space coordinate is: $(-1)^L$;

spin coordinates is: $\xi = 2S - 1$ (for triplet states $= 1$, for singlet $= -1$);

charge coordinates is: $c =$ the eigenvalue of C.

Therefore

$$c\xi(-1)^L = -1\,. \tag{5}$$

Another constant of motion is the spatial parity u, that is the eigenvalue of the operator which reflects space coordinates through the origin. For positronium $u = -(-1)$: (see appendix); since c is also a constant of motion, it follows that ξ *is also a constant of motion* ([4]).

Bound states of positronium are generally not degenerate (outside the trivial J_z degeneracy) and therefore are eigenstates of C and S, but not of L: the orbital momentum is a mixture of L and $L+2$ (for the large components). Since the mixture of L and $L+2$ is irrelevant to our considerations we shall use the ordinary spectroscopic notations for simplicity. By emission of a photon the positronium undergoes transition from the state ${}^{\xi}L$ to the state ${}^{\xi'}L'$ and we must have for these states $c = -c'$ (since a photon is odd under charge conjugation). This selection rule can be written ([14])

$$\xi(-1)^L = -\xi'(-1)^{L'}\,. \tag{6}$$

Consequently, there are lines existing in the corresponding hydrogen spectrum which are absolutly forbidden in the positronium spectrum ([15]):

([13]) For instance: P. A. M. DIRAC: *The principles of quantum mechanics* (3ª ed. Oxford, 1947), p. 78.

([14]) After this work was finished I heard from Prof. GELL-MANN that he has found these rules by perturbation theory some years ago but did not publish them. See also R. FERREL's thesis (Princeton) for the annihilation of positronium.

([15]) Of course, these transitions can be induced by collisions with electrons of surrounding atoms or molecules, since the total charge of the electron field is no longer zero.

such lines are for instance the quadrupole transitions. However electric dipole transitions ($L' = L \pm 1$, $\xi' = \xi$) are allowed and therefore it seems practically impossible to obtain experimental test of this selection rule. Moreover the excluded lines are no longer forbidden when positronium is placed in an external field; indeed c is then no longer a good quantum number since the interaction of the external field is not symmetrical in the two particles of opposite charge.

There is also a selection rule for the decay of the positronium into n photons [4,14], since we must have

$$c = (-1)^n \,. \tag{7}$$

Hence for the lowest state of positronium:

^3S state ($c = -1$) cannot decay into an even number of photons,

^1S state ($c = 1$) cannot decay into an odd number of photons.

It must be noted (and this will be used in 2·4) that there are states of positronium for any given set of values of total angular momentum J and parity u, but not always with both values of c. The non existing sets of values are $J = 0$, $c = -1$ and $c = -u = -(-1)^J$.

Up to now, all solutions of problems in electrodynamics have been obtained by perturbation theory, which allows us to think at all stages of the calculation in terms of particles either real or virtual. The nature of the proofs given above shows that the results obtained apply as well to virtual particles. As it was the case for Furry's theorem, these results can therefore be useful for perturbation calculations since they predict what terms will vanish. For example, in the study of levels in positronium [16] we have a correction to the energy due to the virtual annihilation of the pair. All corrections (including radiative corrections to any order) due to one (or any odd number) quantum virtual annihilation will exist only for states with $c = -1$ and corrections due to two (or any even number) quantum virtual annihilation will exist only in states with $c = 1$. This agrees with published calculations [17,18].

1·4. *Case of other fields in interaction with the electromagnetic field.* – The extension of the preceding section to this case is obvious, but has less physical

(16) This was suggested to me by Dr. B. MOTTELSON.

(17) J. PIRENNE: *Arch. Sci. Phys. Nat.*, **29**, 265 (1947); see also B. V. BERETETSKI: *Journ. Exp. Theor. Phys.*, **19**, 673 (1949) and R. FERRELL's thesis (Princeton, 1952) where one γ virtual annihilation is treated.

(18) R. KARPLUS and A. KLEIN: *Phys. Rev.*, **87**, 849 (1952) where one and two γ annihilations are treated.

interest. For the case of charged fields of spin 1/2, it is sufficient to replace everywhere the word « electron » by proton, or μ-meson, etc.

As for positronium, it must also be noted that for a system of two charge conjugate bosons of spin 0, the excluded values of c are $c = -(-1)^J$ since $J = L$ and since Bose statistics requires $c(-1)^L = 1$. For a system of two charge conjugate spin 1 bosons with non zero rest mass, the only excluded value of c is $c = -1$ for $J = 0$.

2. – Extension to non-electromagnetic charge.

2·1. – The notation of charge can be extended to other couplings. There is no longer charge conservation, except if the theory is invariant under gauge transformations of the first type ([19]), i.e. under the group of transformations $G(\alpha)$, (where α is a real constant), which transform charged field operators according to ([20]):

$$G(\alpha)\psi G^{-1}(\alpha) = \psi e^{i\alpha}\,, \qquad G(\alpha)\psi^* G^{-1}(\alpha) = \psi^* e^{-i\alpha}\,. \tag{8}$$

For charged boson fields we have

$$C\psi C^{-1} = \psi^*\,, \qquad C\psi^* C^{-1} = \psi\,. \tag{9}$$

Without restriction of generality this also holds for fermion fields (i.e. fields described by spinor quantities) if a suitable equivalence transformation is made on them. For Dirac fields, this corresponds to the choice of a Majorana representation ([2]) of Dirac matrices.

The operators $G(\alpha)$ form a group R_2 isomorphic to the group of rotations around an axis. From (8) and (9) one sees that:

$$C^2 = 1\,, \qquad CG(\alpha) = G(-\alpha)C\,. \tag{10}$$

([19]) Unhappily, physicists used the same word « gauge » for two kinds of transformations, but they distinguish between the two of them (see W. PAULI: *Rev. Mod. Phys.*, **13**. 203 (1941)) by calling the one used here « gauge transformation of the first type ». This is a particular case of the second type in which α is a function of space and time coordinates. Transformation of the second type is used when the boson has at least one state of zero mass (K. LE COUTEUR: *Nature*, **165**, 106 (1951)).

([20]) As usual in field theory papers, * means the hermitian conjugate operator and therefore the complex conjugation for ordinary numbers. For spinor fields with several components, the corresponding index will be omitted in this paper, but these components will be considered as a one column matrix (whose elements are operators!). The simbol $\sim$ means transposed.

Therefore C corresponds to a reflexion through a plane containing this axis. In other words, the operators C and $G(\alpha)$ form a group isomorphic to $\mathcal{O}_2$ the real orthogonal group in two dimensions ([21]) (group $C_{\infty v}$ of symmetry of hetero-nuclear molecules). All the non-equivalent irreducible unitary representations of $\mathcal{O}_2$ are known, all are two dimensional except the two $\mathcal{D}_0^{\varepsilon}$

$$\text{(11)} \qquad \mathcal{D}_m \quad \text{with} \quad m > 0 \qquad G(\alpha) \to \begin{pmatrix} e^{im\alpha} & 0 \\ 0 & e^{-im\alpha} \end{pmatrix} \quad C \to \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\text{(11')} \qquad \mathcal{D}_0^{\varepsilon} \quad \text{with} \quad \varepsilon = \pm 1 \quad G(\alpha) \to 1 \qquad\qquad C \to \varepsilon\,.$$

These representations are univalent only for m integer. For simplicity we shall restrict m to be an integer in section **2** and **3**.

The product of such representation is decomposed into irreducible representations according to:

$$\text{(12)} \qquad \mathcal{D}_{m'} \times \mathcal{D}_{m''} = \mathcal{D}_{m'+m''} + \mathcal{D}_{|m'-m''|}\,, \qquad\qquad m' \neq m''$$

$$\text{(12')} \qquad \mathcal{D}_m(\mathrm{I}) \times \mathcal{D}_m(\mathrm{II}) = \mathcal{D}_{2m}(\square\!\square) + \mathcal{D}_0^{+}(\square\!\square) + \mathcal{D}_0^{-}\left(\genfrac{}{}{0pt}{}{\square}{\square}\right),$$

$$\text{(12'')} \qquad \mathcal{D}_m \times \mathcal{D}_0^{\varepsilon} = \mathcal{D}_m\,,$$

$$\text{(12''')} \qquad \mathcal{D}_0^{\varepsilon'}(\mathrm{I}) \times \mathcal{D}_0^{\varepsilon''}(\mathrm{II}) = \mathcal{D}_0^{\varepsilon}(\square\!\square) \qquad \text{with} \quad \varepsilon = \varepsilon'\varepsilon''\,.$$

Les us call I and II the respective variables of the spaces of the representations $\mathcal{D}(\mathrm{I})$ and $\mathcal{D}(\mathrm{II})$. When these two representations have the same m the irreducible representations of $\mathcal{O}_2$ obtained by decomposition of the direct product $\mathcal{D}(\mathrm{I}) \times \mathcal{D}(\mathrm{II})$ are direct sums of equivalent irreducible representations of the permutation group of the two variables I and II ($\square\!\square$ is for symmetric, $\genfrac{}{}{0pt}{}{\square}{\square}$ for antisymmetric). This occurs in (12′) and (12‴). It is important when we deal with two identical particles.

Any state vector belongs to the space of a representation (not irreducible in general). From its physical meaning, the vacuum must belong to the trivial representation, in which each element of the group is represented by the unit matrix. From section **1** we see that for electrodynamics a state with one photon belongs to $\mathcal{D}_0^{-}$ and a state with one electron to an arbitrary but fixed

([21]) Of course reflexion through any plane also represents charge conjugation, and the product of two reflexions through planes making an angle $\beta/2$ with each other will multiply the field operators by the phase $e^{i\beta}$. What is not ambiguous however is the correspondence between charge conjugation, gauge invariance on one hand and the linear unitary representations of group $\mathcal{O}_2$ on the other hand.

representation $\mathcal{D}_m$, the two dimensions corresponding to the two independant charge conjugate states: one positon or one negaton. More generally a state vector which describes a system of p photons and q electrons is a vector in the space of the representation $(\mathcal{D}^-)^p \times (\mathcal{D}_m)^q$ and this vector is fixed as long as the system is isolated.

If we had taken in account charge conservation only, we would have dealt with the one dimensional representations d_m (with m integer $\gtrless 0$) of the gauge invariance group R_2. Irreducible representations of $\mathcal{O}_2$ are generally reducible representations of its subgroup R_2; the decomposition is according to

$$\mathcal{D}_m \to d_m + d_{-m}\,, \qquad (m > 0) \tag{13}$$

$$\mathcal{D}_0^\varepsilon \to d_0\,. \tag{13'}$$

Different irreducible representations of $\mathcal{O}_2$ give different representations of R except $\mathcal{D}_0^+$ and $\mathcal{D}_0^-$ which both give d_0. Therefore it is only for states of tota charge zero that we shall have selection rules due to charge conjugation and not to charge conservation. These selection rules are imposed by the conservation of $\varepsilon = c$ in the representations $\mathcal{D}_0^\varepsilon$.

2·2. *Case of neutral boson fields coupled with a Dirac field.* – As an example of application of the general considerations given in **2·1** we now study the case of neutral boson fields coupled with a Dirac field. For simplicity we deal only with the customary meson theories (see for instance KEMMER ([9]). More precisely we consider only couplings linear in spin 0 or spin 1 boson fields or their first derivatives, and which do not contain derivatives of the Dirac. As is well known these couplings are of the form $B\cdot J$ where B is either one of the four boson fields or its gradiant (in four dimensions); the « dot » indicates a scalar product and J is one of the five invariants (scalar s, vector v, skew tensor t, pseudovector a and pseudoscalar p) that one can form with a Dirac field and its conjugate ([22]):

$$J_i = \frac{1}{2} f[\tilde{\psi}^* F_i \psi - \tilde{\psi} \tilde{F}_i \psi^*]\,; \qquad i,\ 1 \text{ to } 5\,. \tag{14}$$

The F_i are 1, 4, 6, 4, 1 linear independant four row four column matrices which, in the Majorana representation ([2]), are either symmetric or skew-

([22]) More loosely J_i is often written $\tilde{\psi}^* F_i \psi$ since $\tilde{\psi}^* F_i \psi = -\tilde{\psi}\tilde{F}_i\ \psi^*$ when $\tilde{\psi}^*\psi + \tilde{\psi}\psi^* = 0$.

symmetric ([23]):

$$(15)\qquad \widetilde{F}_i = -\theta_i F_i \quad \text{with} \quad \theta_i = 1,\ -1,\ -1,\ 1,\ 1\,.$$

Thus from (9) we find that:

$$(16)\qquad CJ_iC^{-1} = \frac{1}{2} f[\widetilde{\psi}F_i\psi^* - \widetilde{\psi}^*\widetilde{F}_i\psi] = \frac{\theta_i}{2} f[-\widetilde{\psi}\widetilde{F}_i\psi^* + \widetilde{\psi}^*F_i\psi] = \theta_i J_i\,,$$

so that the charge conjugation of the interaction $B\cdot J$ requires

$$(17)\qquad CB_iC^{-1} = \theta_i B\,.$$

The eight types of coupling will be denoted here (as in ([7])) by *Ss*, *Vv Aa Pp* for those which do not involve derivatives of the boson field and by *Sv*, *Vt*, *At*, *Pa* for the others. The coupling constants f are now the charges of the fermions. The J's are invariant under gauge transformation (see ([8]) and ([14])) as are the B's since they describe « strictly neutral » bosons; therefore there is charge conservation. Moreover (17) shows that:

Pp, *Pa*, *Ss* and *Aa* bosons belong to $\mathcal{D}_0^+$, since then $\theta_i = 1$,

Vv, *Vt*, *Sv* and *At* bosons belong to $\mathcal{D}_0^-$, since then $\theta_i = -1$.

S and *A* bosons with *both* types of couplings are not described by irreducible representations and therefore do not lead to selection rules for charge conjugation; consequently *S* or *A* bosons considered in the following will have only one type of coupling.

The extension of Furry's theorem (made by FURRY himself ([3])) is then straightforward:

THEOREM 1. A reaction between neutral bosons through one intermediary fermion field is forbidden if the total number of v and t couplings is odd ([24]).

([23]) The notations used here are those of reference ([7]). For the equation (15) see W. PAULI: *Ann. Inst. H. Poincaré*, **6**, 109 (1936), where the corresponding property for an arbitrary hermitian representation of Dirac matrices is proved. In Majorana representations, the matrix C of PAULI's paper is $= 1$.

([24]) V and P bosons can have both their coupling terms; evidently *Vv* and *Vt* or *Pp* and *Pa* couplings of the same meson are only counted as one.

Indeed this reaction is represented by

$$\prod_i \mathcal{D}_0^{\varepsilon_i} = \prod_f \mathcal{D}_0^{\varepsilon_f}\,, \tag{18}$$

which implies $\prod_a \varepsilon_a = 1$, where i = initial, f = final and a = all. The most interesting case of application is the spontaneous decay of a boson (the other selection rules occuring in this case are listed in the appendix). For instance π^0 mesons (P bosons) cannot decay into an odd number of photons.

The extension of **1·3** to system of one fermion and its antiparticle is also straightforward; as before we have $c = -\xi(-1)^L$ and c is a good quantum number. The production of bosons by collision of a fermion with its antiparticle requires that $c' = c$ for P or Ss or Aa bosons, $c' = -c$ for V or At or Sv bosons. States with $c = 1$ cannot decay into an odd number of V, A or St bosons. States with $c = -1$ cannot decay into an even number of Vt At, St bosons and/or any number of P, Ss, Aa bosons (other selection rules for annihilation are found in appendix).

2·3. *Cases with no conservation of charge.* – Of course charge conjugation can also be studied in theories without charge conservation. But it is then trivial. For instance, let us suppose that in **2·2** the fermion field is a field of MAJORANA ([2]) (i.e. « strictly neutral ») particles. Then $\psi = \psi^*$ and (14) shows that $J_2 = J_3 = 0$. The theory is invariant under the group of two elements 1, C which has two representations: $c = \pm 1$. Each pair of fermions belong to $c = 1$ and can annihilate into any number of bosons. The same applies to the interaction of **2·2** where, in a Majorana representation,

$$J_i = \frac{1}{2} f[\tilde{\psi} F_i \psi + \tilde{\psi}^* F_i \psi^*]\,, \tag{19}$$

$J_2 = J_3 = 0$. Here there is no charge conservation and only pairs of identical fermions can annihilate.

2·4. *Most general case, with several kinds of charges.* – The study of charge conjugation can be extended to any kind of couplings in present field theories (see also ([7])). Here we want to study the most general case. Indeed a charged particle can be neutral for some charges: for example, electromagnetic-charged mesons have no nuclear charge. We need to consider only the charges which are conserved. To each kind of charge corresponds a gauge group. Let us call $\mathcal{G}$ the direct product of all these gauge groups; the representations $d_{m_1, m_2, m_3, \ldots}$ (with $m_i \gtrless 0$) are one-dimensional. Let us add to $\mathcal{G}$ the (unique) operation of charge conjugation. The new group has two dimensional representations $\mathcal{D}_{m_1,m_2,m_3,...}$ (with $m_i > 0$) and two one dimensional representations $\mathcal{D}^{\varepsilon}_{0,0,0,...}$ (with $\varepsilon = \pm 1$), to which belong self-charge conjugate systems. The extensions of the considerations of section **2·1** to this section **2·4** is obvious. Therefore selection rules due to charge conjugation will appear only for self-charge conjugate systems and those are very easy to select. When they do not contain an odd number of any kind of « strictly neutral » particles, no knowledge of the nature of the couplings is necessary for the determination of their representation. For self-charge conjugate systems containing only one pair of charged particles, we have already foundthe list of forbidden values of c for spin 1/2 (in **1·3**) and spin 0 and 1 (in **1·4**), since this does not depend on the nature of the charge.

For instance, from this there results the following rule: An Sv meson cannot decay into a positon-negaton pair [4,25] or into a pair of conjugated charged mesons of spin 0 (as $\pi^+ + \pi^-$) or spin 1 [26].

3. – Charge symmetry.

3·1. *Charge symmetry and charge conjugation.* – Charge symmetry is the hypothesis that the formalism is invariant under exchange of protons and neutrons. It seems to be a reasonable hypothesis for then it implies the equality of the specific nuclear nn and pp forces. Let us call N the operator exchanging p and n; it also exchanges $\tilde{p}$ (antiproton) and $\tilde{n}$ (antineutron). We have $N^2 = 1$ and it is very easy to see from (8) and (9) that N commutes with C and $G(\alpha)$. Therefore N, C and $G(\alpha)$ form a group isomorphic to the direct product $(1 + N) \times \mathcal{O}_2$, isomorphic to the group $D_{\infty v}$ of symmetry of homonuclear diatomic molecules. Its irreducible representations are all given by the direct product of those of $\mathcal{O}_2$ and $1 + N$. The group $1 + N$ has only two representations labelled η (in them N is represented by η). The representation of $D_{\infty v}$ will then be noted as those of $\mathcal{O}_2$ but with η as left superscript. For instance:

$$ {}^{\eta'}\mathcal{D}^{\varepsilon'}_0 \times {}^{\eta''}\mathcal{D}^{\varepsilon''} = {}^{\eta}\mathcal{D}^{\varepsilon}_0 \qquad \text{with } \eta = \eta'\eta'' , \quad \varepsilon = \varepsilon'\varepsilon'' . \tag{20} $$

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[25] This does not agree with the calculations of J. STEINBERGER: *Phys. Rev.*, **76**, 1180 (1949). I heard from Dr. H. FUKUDA that he has proved this selection rule by the use of the equivalence theorem.

[26] Charge conjugation also forbids the decay of a Aa meson into $\pi^+ + \pi^-$ but this is forbidden by parity conservation alone. Similarly, the annihilation into $\pi^+ + \pi^-$ is forbidden by charge conjugation for systems of proton-antiproton or neutron-antineutron with $\xi = 1$, $J = L$, but conservation of parity alone forbids such annihilation for $J = L$.

3·2. *Extensions of Furry's theorem.*

3·2.1 *Neutral mesons.* – We shall [23] write φ the invariant constructed from the meson fields themselves or the first order derivatives, J_{in} those made from the neutron field and J_{ip} those from the proton field. The interaction between a neutral meson field and the nucleons is

$$H_{0i} = \varphi_i(f_{in}J_{in} + f_{ip}J_{ip}) = \varphi_i[f_{i0}(J_{in} + J_{ip}) + f_{i3}(J_{in} - J_{ip})\,, \tag{21}$$

with the following relation between the coupling constants [27]

$$f_3 = \frac{1}{2}(f_n - f_p)\,, \qquad f_0 = \frac{1}{2}(f_n + f_p)\,. \tag{22}$$

The effect of the operator N is easily seen to be $NJ_nN^{-1} = J_p$, $NJ_pN^{-1} = J_n$ and hence mesons with pure f_0 coupling correspond to the ${}^{+}\mathcal{D}_0^{\theta_i}$ representation and those with pure f_3 coupling to the ${}^{-}\mathcal{D}_0^{\theta_i}$. When one considers only these two kinds of neutral mesons, one gets the following extension of Furry's theorem [7,8]: (see [18])

$$\prod_i {}^{\eta_i}\mathcal{D}_0^{\varepsilon_i} = \prod_f {}^{\eta_f}\mathcal{D}_0^{\varepsilon_f}\,, \tag{23}$$

or

$$\prod_a \eta_a = 1 \quad and \quad \prod_a \varepsilon_a = 1\,. \tag{24}$$

Hence, THEOREM 2. A reaction between neutral mesons through the nucleon field is forbidden if *either* the number of f_3 couplings is odd, *or* the number of v and t couplings is odd.

Photons [28] have $f_n = 0$, therefore $f_0 = -f_3 = e/2$, thus they belong to the reducible representation ${}^{+}\mathcal{D}_0^{-} + {}^{-}\mathcal{D}_0^{-}$; they are an example of non charged symmetric mesons. With such mesons we see that theorem 2 reduces indeed to theorem 1.

3·2.2. *Charged mesons.* – Only nucleons have nuclear charge: charged mesons belong to the $\mathcal{D}_0$ representations but, of course, they are not self-charge conju-

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([27]) Index 0 and 3 are chosen here in accordance with isotopic spin formalism: see **4·1**.

([28]) This is in accord with the current views that anomalous magnetic moments of nucleons are not due to direct interaction with magnetic fields but result from radiative processes through the charged meson field.

gate. The interaction between charged meson and nucleon fields is

$$H_{ci} = f_i(\varphi_i^* \cdot \tilde{\psi}_n^* F_i \psi_p + \varphi_i \cdot \tilde{\psi}_p^* F_i \psi_n). \tag{25}$$

For convenience, and without lack of generality, neutron and proton fields are made anticommuting. From $CH_{ci}C^{-1} = H_{ci}$ we can verify that

$$C\varphi_1 C^{-1} = \theta_i \varphi_i^* \; ; \qquad C\varphi_i^* C^{-1} = \theta_i \varphi_i \, . \tag{26}$$

We also have

$$NH_{ci}N^{-1} = f_i(N\varphi_i^* N^{-1} \cdot \tilde{\psi}_p^* F_i \psi_n + N\varphi_i N^{-1} \cdot \tilde{\psi}_n^* F_i \psi_p), \tag{27}$$

therefore

$$N\varphi_i N^{-1} = \varphi_i^* \, , \qquad N\varphi_i^* N^{-1} = \varphi_i \, .$$

Equation (26) and (28) can also be written

$$C(\varphi_i \pm \varphi_i^*)C^{-1} = \pm \theta_i(\varphi_i \pm \varphi_i^*) \, , \qquad N(\varphi_i \pm \varphi_i^*)N^{-1} = \pm (\varphi_i \pm \varphi_i^*) \, . \tag{29}$$

Equation (29) means that $\varphi_i + \varphi_i^*$ belong to the ${}^{+}\mathcal{D}_0^{\theta_i}$ representation and $\varphi_1 - \varphi_i$ to the ${}^{-}\mathcal{D}_0^{-\theta_i}$ representation. Therefore a given charge conjugate state of a charged meson is represented by the reducible representation: ${}^{+}\mathcal{D}_0^{\theta_i} + {}^{-}\mathcal{D}_0^{-\theta_i} = \sum_K {}^{K}\mathcal{D}_0^{K\theta_i}$ with $K = \pm 1$. We can now prove the following theorem [5,6]

Theorem 3. A reaction between charged and neutral mesons through the nucleon field is forbidden if the total number of f_3, v and t couplings is odd:

Indeed such a reaction is represented by

$$\prod_{i0} {}^{\eta_{0i}}\mathcal{D}_0^{\varepsilon_{0i}} \prod_{ic} \left(\sum_K {}^{K_i}\mathcal{D}_0^{K\varepsilon_{ci}}\right) = \prod_{f0} {}^{\eta_{0f}}\mathcal{D}_0^{\varepsilon_{0f}} \prod_{fc} \left(\sum_K {}^{K_f}\mathcal{D}_0^{K_f \varepsilon_{cf}}\right) . \tag{30}$$

Due to (20) this requires that there is at least one set of K_a such that

$$\prod_a \eta_{0a} K_a = 1 \quad \text{and} \quad \prod_a \varepsilon_{0a} K_a \varepsilon_{ca} = 1 \, , \tag{31}$$

or by eliminating the set of K_a's, we find the following condition,

$$\prod_a \eta_{0a} \varepsilon_{0a} \varepsilon_{ca} = 1 \, . \tag{32}$$

3·3. *Annihilation of nucleon antinucleon pairs.* – From the definition of N one sees that $\psi_n \pm \psi_p$, $\psi_n^* \pm \psi_p^*$ belong to the eigenvalue $\eta = \pm 1$ of N and to the two dimensional representation ${}^{\eta}\mathcal{D}_m$. For a system of a nucleon and antinucleon one readily sees that:

neutral systems, $n\tilde{n}$ or $p\tilde{p}$, belong to ${}^{+}\mathcal{D}_0^{\varepsilon} + {}^{-}\mathcal{D}_0^{\varepsilon}$ with $\varepsilon = -\xi(-1)^L$,

charged systems, $n\tilde{p}$ or $p\tilde{n}$, belong to ${}^{+}\mathcal{D}_0^{\varepsilon} + {}^{-}\mathcal{D}_0^{-\varepsilon}$ with $\varepsilon = -\xi(-1)^L$.

Hence the selection rules for emission of a meson by collision of a nucleon and an antinucleon only exist for neutral mesons emitted by neutral systems and are those of **1**·4.

From **2**·2 where the representations of mesons and photons are given, it is easy to see the corresponding rules for annihilation and this is left to the reader (see also the appendix for absolute selection rules due to angular momentum and parity conservation). As an example, in table I, the possible annihilations into photons and/or π-mesons (considered as pseudoscalar symmetric [29]) are given for S-states [30]

TABLE I. – *Possible and forbidden modes of annihilation of an antinucleon under the consideration of conservation of angular momentum, conservation of parity, charge conjugation and charge symmetry.*

Systems	States	Allowed into 2 particles	Allowed into 3 particles	Forbidden into
$n\tilde{n}$ or $p\tilde{p}$	^3S	$\pi^0 + \gamma$ $\pi^+ + \pi^-$	3γ $\gamma + 2\pi^0$ $\gamma + \pi^+ + \pi^-$ $\pi^0 + \pi^+ + \pi^-$	any number of π^0 and/or an even number of γ
	^1S	2γ	$3\pi^0$ $2\gamma + \pi^0$ $\pi^0 + \pi^+ + \pi^-$	an odd number of γ and zero or any number of π^0
$n\tilde{p}$ or $p\tilde{n}$	^3S	$\pi^{\pm} + \pi^0$ $\pi^{\pm} + \gamma$	$\pi^{\pm} + 2\gamma$ $\pi^{\pm} + \gamma + \pi^0$	charged π and zero or any even number of π^0
	^1S		$2\pi^{\pm} + \pi^{\pm}$ $\pi^{\pm} + 2\pi^0$ $\pi^{\pm} + \pi^0 + \gamma$ $\pi^{\pm} + 2\gamma$	an even number of π mesons only

(29) N. KEMMER: *Proc. Camb. Phil. Soc.*, **34**, 354 (1938).

(30) As an example of study for the construction of table I, we give here the re-

3·4. *Remarks.*

3·4.1. *Selection rules due to charge symmetry only.* – There are also selection rules due only to the conservation of η, the eigenvalue of N. They will appear for self-charge symmetric states. Such selection rules, due to charge symmetry only, and not to charge conjugation, do not belong properly to the subject of this paper. See however **4**·2.

3·4.2. *Validity of selection rules due to charge symmetry.* – These selection rules are not absolute since they depend on the neglect of the electromagnetic interactions. However, electromagnetic coupling is weak compared to nuclear coupling and the selection rules due to charge conjugation combined with charge symmetry are good approximations. The reader is referred to the paper of PAIS and JOST [8] for an interesting discussion on this point.

3·4.3. *Extension to other couplings.* – The study of charge symmetry combined with charge conjugation can be extended to other kinds of couplings involving the nucleons. As an example the reader is referred to a previous work of the author [7], where an extension of Furry's theorem is established for direct coupling between a pair of nucleons and another pair of fermions as in the Fermi theory of β-radioactivity.

4. – Isotopic formalism and charge conjugation.

4·1. *Charge independence.* – The use of the isotopic spin formalism is well known and here only results without proofs will be quoted. Neutrons and protons are considered as states of the same particle, the nucleon. These states are labelled by a two valued index: i.e. the nucleon field is represented as a two line matrix $\psi = \begin{pmatrix}\psi_n \\ \psi_r\end{pmatrix}$: We shall use in the following the well known Pauli matrices

$$\tau_0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \qquad \tau_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \qquad \tau_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \qquad \tau_3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}.$$

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presentations corresponding to a system of two charged mesons:

$$\Big(\sum_K {}^K\mathcal{D}_0^{K\varepsilon}(\mathrm{I})\Big)\times\Big(\sum_K {}^K\mathcal{D}_0^{K\varepsilon}(\mathrm{II})\Big) = 2\,{}^+\mathcal{D}_0^+(\square\square) + {}^-\mathcal{D}_0^-(\square\square) + {}^-\mathcal{D}_0^-\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}\right)$$

and it is easily seen that systems of charge 2 (as $2\pi^+$ or $2\pi^-$) belong to ${}^+\mathcal{D}_0^+(\square\square) + {}^-\mathcal{D}_0^-(\square\square)$ (it must be of course $\square\square$) and the self-charge conjugate systems $(\pi^+ + \pi^-)$ belong either to ${}^+\mathcal{D}_0^+(\square\square)$ (then $c = \eta = 1$ and $J = L$ is even, $u = 1$) or to ${}^-\mathcal{D}^-\left(\begin{smallmatrix}\square\\ \square\end{smallmatrix}\right)$ (then $c = \eta = -1 = u$, $J = L$ is odd).

They satisfy

(33) $$\tilde{\tau}_i = \tau_i^* \qquad \text{with} \quad i = 0 \text{ to } 3 .$$

Charge independance is obtained by invariance under the group R_3 of rotations in the three dimensional isotopic spin space. These rotation of an angle α around an axis whose orientation is given by the unit vector $\boldsymbol{n}$ will be denoted by: $R(\alpha, \boldsymbol{n})$. The nucleon has an isotopic spin 1/2, i.e. $R(\alpha, \boldsymbol{n})$ induces on ψ and ψ^* the linear transformations

(34) $$R(\alpha, \boldsymbol{n})\psi R^{-1}(\alpha, \boldsymbol{n}) = \exp[i(\alpha/2)\boldsymbol{\tau}, \boldsymbol{n}]\psi = \cos\alpha/2 + i\boldsymbol{\tau}\cdot\boldsymbol{n}\,(\sin\alpha/2)\psi ,$$

(34′) $$R(\alpha, \boldsymbol{n})\psi^* R^{-1}(\alpha, \boldsymbol{n}) = \exp[-i(\alpha/2)\boldsymbol{\tau}^*, \boldsymbol{n}]\psi^* = (\tilde{\psi}^* \exp[-i(\alpha/2)\boldsymbol{\tau}, \boldsymbol{n}])_{\text{transposed}}$$

where $\boldsymbol{\tau}$ means τ_1, τ_2, τ_3.

With ψ and its conjugate one can expect to form two quantities (see (36)):

$$D_{1/2} \times D_{1/2} = D_0 \times D_1 ,$$

of isotopic spin $T = 0$ and $T = 1$; they are $\tilde{\psi}^*\psi$ for $T = 0$ and $\tilde{\psi}^*\boldsymbol{\tau}\psi$ for $T = 1$ Hence there are two possible meson theories giving charge independent forces; as well known the case with $T = 0$ corresponds to a neutral meson theory with coupling f_0; the case $T = 1$ corresponds to the symmetrical ([29]) meson theories where the meson has three isotopic states of electric charge -1, 0, 1. The couplings are for both theories:

(35) $$H = \sum_r f_r \varphi_{ir}[\tilde{\psi}^* \tau_r F_i \psi - \tilde{\psi}\tilde{\tau}_r \tilde{F}_i \psi^*] = \sum \varphi_{ir} J_{ir} ,$$

with $r = 0$ for $T = 0$ and $r = 1$ to 3, $f_1 = f_2 = f_3$ for $T = 1$.

4·2. *Charge symmetry*. – We could have treated charge symmetry as we treated charge independence by the invariance under a group isomorphic to $\mathcal{O}_2$. Indeed this can be done in the isotopic spin formalism. Charge independence which is obtained by invariance under a R_3 group is a particular case of charge symmetry which is obtained by the invariance under the subgroup $\mathcal{O}_2$ of R_3, isomorphic to the group composed of the rotations around the third axis of the isotopic space and the reflexions through planes containing this axis.

The irreducible representations of R are denoted by D_J (with $2J$ integer $\geqslant 0$ and the reduction of their products into irreducible representations is given by

(36) $$D_J \times D_{J'} = D_{J+J'} + D_{J+J'-1} + \dots + D_{|J-J'|}.$$

They are generally reducible representations of the subgroup $\mathcal{O}_2$ of R_3, and their decomposition is given by

(37) $$D_J \to \mathcal{D}_{1/2} + \mathcal{D}_{3/2} + \ldots + \mathcal{D}_J \qquad \text{for } J \text{ half integer },$$

(37′) $$D_J \to \mathcal{D}_0^{(-1)^J} + \mathcal{D}_1 + \ldots + \mathcal{D}_J \qquad \text{for } J \text{ integer}.$$

Since here charge symmetry is described by the invariance under the same group, $\mathcal{O}_2$, as is used in section 2 for the study of charge conjugation, the consideration of section 2 can be applied here. Here selection rules due to charge symmetry will only appear for self charge symmetrics-states, i.e. states with the same number of p and n (or $\tilde{p}$ and $\tilde{n}$!) and/or any number of neutral mesons (even if S or A mesons have both types of couplings) and pair of conjugate charged mesons. The most interesting case for the application of such rules are nuclear reactions with such nuclei as ${}_1H^2$, ${}_2He^4$, ${}_3Li^6$, ${}_4Be^8$, ${}_5B^{10}$, ${}_6C^{12}$ ${}_7N^{14}$, ${}_8O^{16}$ .... Each level of these nuclei is an eigenvector of the operator N (exchange of p and n) and the eigenvalue η is a good quantum number. Indeed equation (37′) shows that $\eta = (-1)^T$, where T is the total isotopic spin of the nucleus when the hypothesis of charge independant forces is made. For instance, the reaction ${}_8O^{16}(d, \alpha){}_7N^{14*}$ is described by

(38) $$\begin{cases} {}_8O^{16} + {}_1H^2 \to {}_2He^4 + {}_7N^{14*} , \\ \mathcal{D}_0^+ \times \mathcal{D}_0^+ \to \mathcal{D}_0^+ \times \mathcal{D}_0^\eta \end{cases}$$

since the ground state of all these nuclei has $T = 0$. Therefore $\eta = 1$ and excited states of ${}_7N^{14*}$ with $\eta = -1$ cannot appear in this reaction, e.g. first excited state. This has been recently emphasized by KROLL (31).

4·3. *Charge conjugation.* – We can still define C by (9), where now ψ has a two value isotopic index. On the explicit value of the τ_i matrices we can verify that

(39) $$\tau_2\tilde{\tau}_i = \tau_2\tau_i^* = \zeta_i\tau_i\tau_2 \qquad \text{with} \quad \zeta_i = 1; -1, -1, -1 .$$

Let us consider the following rotation in isotopic space $M = R(\pi, \boldsymbol{n}_2)$ where $\boldsymbol{n}_2$ has components (0, 1, 0). We note that C and M commute: $CM = MC = U$ with

(40) $$U\psi U^{-1} = i\tau_2\psi^* , \qquad U\psi^* U^{-1} = i\tau_2\psi .$$

(31) I heard it from Prof. WEISSKOPF at the summer school des Houches. See also the quotation of K. K. ADAIR: *Phys. Rev.*, **87**, 1041 (1952), in a foot note.

Let R be an arbitrary rotation $R(\alpha, \boldsymbol{n})$ in isotopic space; we have

$$(41)\quad UR\psi R^{-1}U^{-1} = i\tau_2 \exp[i(\alpha/2)\boldsymbol{\tau}\cdot\boldsymbol{n}]\psi^*, \; RU\psi U^{-1}R^{-1} = i \exp[-i(\alpha/2)\boldsymbol{\tau}^*, \boldsymbol{n}]\tau_2\psi^*$$

from (39)

$$(42)\qquad RU\psi U^{-1}R^{-1} = i\tau_2 \exp\left[i(\alpha/2)\boldsymbol{\tau}, \boldsymbol{n}\right]\psi^* = UR\psi R^{-1}U^{-1}\,.$$

Similarly

$$(43)\qquad UR\psi^* R^{-1}U^{-1} = i \exp\left[i(\alpha/2)\boldsymbol{\tau}\cdot\boldsymbol{n}\right]\tau_2\psi = RU\psi^* U^{-1}R^{-1}\,.$$

Therefore U commutes with every $R(\alpha, \boldsymbol{n})$. The group generated by rotations in isotopic space and charge conjugation is therefore isomorphic to the group $\mathcal{O}_3$, i.e. the direct product $(1 + U)\times R_3$. That is the group of rotations and reflexions in isotopic space.

The irreducible representations of this group are the direct product of those of R_3 and those of the group of two elements. They are denoted by D_J^ε with $\varepsilon = \pm 1$. The case with $\varepsilon = -1$ corresponds to the « pseudo » quantities. With the convention of this paragraph, U, the inversion through the origin in isotopic space has the eigenvalue $t = \varepsilon(-1)^J$. This quantity is the « parity » in isotopic space.

Representations of the mesons. – The invariance of the interaction (see (35)) with respect to U gives:

$$(44)\qquad UHU^{-1} = f_{ir}U\varphi_{ir}U^{-1}[\tilde{\psi} i\tilde{\tau}_2\tau_r i\tau_2 F_i\psi^* - \tilde{\psi}^* i\tilde{\tau}_2\tilde{\tau}_r i\tau_2\tilde{F}_i\psi]\,,$$

and from (15) and (39)

$$(44')\qquad UHU^{-1} = \zeta_r\theta_i U\varphi_{ir}U^{-1}J_{ir}\,.$$

Therefore

$$(45)\qquad U\varphi_{ir}U^{-1} = \theta_i\zeta_r\varphi_{ir}\,.$$

For mesons $t = \theta_i\zeta_r$; hence the value of the ε of their representation. Mesons with $T = 0$ belong to the $D_0^{\theta_i}$ representation and mesons with $T = 1$ belong to $D_1^{\theta_i}$.

For instance, the pseudoscalar symmetric mesons (as very likely π-mesons are) belong to D_1^+.

Representations of the nucleons. – Nucleons belong to the reducible representation $D_{1/2}^+ + D_{1/2}^-$.

4·4. *Selection rules specific to charge conjugation and charge independence.* – The only new selection rules due to charge conjugation, which can be expected by restricting the hypothesis of charge symmetry to charge independence, will arise from the identification of corresponding charged and neutral mesons as different isotopic states of the *same* particle.

For instance, if the isotopic spin is a good quantum number in the reaction ([32])

$$\zeta^{\pm} \to \pi^{\pm} + \pi^{0}\,, \tag{46}$$

it is easy to see that $\zeta^{\pm}$ must be a vector meson. The possibility of $\zeta^{\pm}$ being a *Sv* meson, which was allowed by charge symmetry is now forbidden by charge independence.

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APPENDIX

We restrict ourselves ([33]) to systems composed of particles of spin 0, 1/2, and 1; the case of particles of spin 1 with mass zero (γ) must be considered separatly. The total angular momentum J and the parity u are given for the center of mass frame of reference.

Conservation of the angular momentum.

One spin 0 particle + one γ . . .	$J \neq 0$, see for instance ([36])
Two γ	$J \neq 1$ ([34,35])
Two identical spin 0 particles . .	J even, see for instance ([36])

([32]) F. C. POWELL at the Copenhagen conference, June 1952.

([33]) The complete list for particles of any spin is given in ([37]) and notes with G. BONNEVAY to appear shortly in *C. R. Acad. Sci. Paris.*

([34]) L. D. LANDAU: *Dokl. Akad. Nauk. SSSR*, **60**, 207 (1948); also E. WIGNER quoted by J. STEINBERGER: *Phys. Rev.*, **76**, 1180 (1949).

([35]) C. N. YANG: *Phys. Rev.*, **77**, 242 (1950).

([36]) D. C. PEASLEE: *Helv. Phys. Acta*, **23**, 845 (1950).

Conservation of parity.

Particle	S = scalar,	V = vectorial,	A = pseudovectorial,	P = ps. sc.
J	0	1	1	0
u	+	−	+	−

Two particles of spin 0	$u = (-1)^J u_1 u_2$, (36)
One of spin 0, one of spin 1 . .	if $J = 0$, only $u = -u_1 u_2$, (37)
Two γ	if J odd, only $u = +1$ (34,35)
Three particles of spin 0	if $J = 0$, only $u = u_1 u_2 u_3$ (37)

Note that for a system of one Dirac particle and its antiparticle, all sets of J, u are allowed, but $u = -(-1)^L$, where L and/or $L+2$ is the relative orbital momentum of the large components (35,37).

(37) L. MICHEL: *Compt. Rend. Acad. Sci. Paris*, **234**, 703 and 2161 (1951).

RIASSUNTO (*)

L'autore fa uno studio sistematico sull'invarianza delle teorie dei campi rispetto alla coniugazione delle cariche, allo scopo di indicare *tutte* le possibili regole di selezione derivanti dalla coniugazione delle cariche, da sola o combinata con la simmetria o l'indipendenza delle cariche. Si dimostra che usando il formalismo di spin isotopico l'invarianza rispetto alla coniugazione delle cariche corrisponde alla conservazione della parità isotopica.

(*) *Traduzione a cura della Redazione.*