Chapter 2

Commentary

The present volume includes about a tenth of the scientific heritage of Louis Michel which spans the second half of the 20th century. The aim of this commentary is to place the selected papers within his oeuvre indicating at the same time their relations to present day trends.

We begin with a general remark. Louis Michel started his carrier as an experimental particle physicist, but early on he demonstrated a predilection for a theoretical comprehension of the subject and, in particular, for an understanding of the underlying symmetry principles. Not satisfied with borrowing ready prescriptions he studied the group theoretical background seriously. He strove to speak mathematics in the language of mathematicians keeping in mind at the same time the problems of experimental physics. In the late fifties he had a joint work with a leading experimental physicist, Valentine Telegdi; in 1971 his mathematical contributions to the Comptes Rendus of the French Academy were presented by no less an authority than Henri Cartan. Louis liked to quote Galileo's words: "The great book of the Universe stays open before our eyes; but in order to understand it, we have first to learn the language in which it is written: the mathematics." (see [XLIII], and [138], p. 8). He liked both physics and mathematics (being less appreciative of what people used to call "mathematical physics"). In the words of his long time friend and co-author Kameshwar Wali:

Like C.P. Snow[1]*, Louis is a man of two cultures, the culture of pure mathematics and the culture of theoretical physics. He moves freely between the two, fraternizing with both pure mathematicians and the physicists, bringing enlightenment to the two sides, which are as far apart as the scientists and the literary intellectuals that Snow talked about in 1959.* [W94]

This has set high demands, especially to the physicist reader of his articles but it provides a great advantage: mathematical results derived originally for problems of particle physics were later effortlessly applied (in particular, by the author) to other domains and problems.

[1]Charles Percy Snow (1905-1980), English chemist and novelist, lamented in his famous lecture *The Two Cultures* of 1959 the gulf between scientists and "literary intellectuals".

1. Early work on particle physics [2, 7, 12, 15, 19, 23]

Moving from experimental to theoretical physics (after an inspiring seminar by Powell and Occhialini at Blackett's laboratory in Manchester) the 25-year old Louis Michel did not waste his time. In the pre-Internet era participation in authoritative conferences was particularly important and the young man tried not to miss opportunities (even if that required hitchhiking when he could not afford the train ticket). His start was spectacular. His first paper [1] in response to a question posed by Christian Møller at the 1948 Bristol conference contains what came to be known as *Michel's parameters* and was published in the prestigious British journal *Nature*. His subsequent more detailed exposé on the subject[2] **[2]** is, in fact, better known. At his next conference (in Edinburgh, November, 1949), Michel realized that he had understood something which was not clear to leading physicists, not even to his revered mentor, Leon Rosenfeld: baryon number conservation. A few months later he published his second article in Nature, [3] (where baryon number is called *mesic charge*).

Publications in Physical Review of 1949 (cited in [3]) propose to look for antiprotons in beta decay processes. Experimentalists report at an authoritative conference that they have not seen any. Michel explains that they should not have expected to find an antibaryon in such a process. His argument is simple and elegant. Processes in which two nucleons go into a pion, even if permitted by charge conservation - like

$$p + n \to \pi^+, \tag{2.1}$$

are excluded by the observed stability of atomic nuclei. But, the argument continues, symmetries of particle interactions tell us that if a process is forbidden a whole class of related processes is also forbidden. In particular, the fact that the process (2.1) is not allowed implies that the reaction $n + \pi^- \to p^-$, obtained from it by replacing a particle on the left hand of the equation with its antiparticle in the right hand side and vice versa is also forbidden. (Michel uses the sign p^-, the "negative proton", for an antiproton). His conclusion is that an antiproton can only be created in a pair with a nucleon (so that the total baryon number is not changed in the process). The argument also implies that there could not be antinucleons in a stable nucleus. Note that in the same two-page article Michel discusses the case of Majorana spinors which describe particles coinciding with their "charge conjugate" antiparticles - a notion which only became popular among theorists nearly forty years later in discussing neutrino oscillations. The story of uncovering the baryon conservation law is told by Michel (suppressing his own role!) in **[XXX]**.

Considering decay processes Michel was not satisfied to write down a convenient parametric expression for the decay amplitude. He looked for the conceptual framework and the appropriate mathematical background allowing to understand and find the true place of the phenomenological formula, an approach that became a trademark of his scientific style. It is at this early stage (before meeting his model-guide Eugene Wigner!) that Michel appreciated the role of symmetry as a super law underlying physical models and theories. He exploited in [4] the permutation symmetry between the four fermions extending the domain of application of his analysis to nucleon-nucleon scattering (cf. [6, 5]). Mastering the mathematical

[2] We refer to papers reprinted in this volume in boldface. Ref. **[2]** is cited over 350 times.

theory of symmetry groups was not an excuse for Michel to forget that physics is first and foremost an experimental science. Analyzing current experimental data on the decay of a heavy lepton he assumed (correctly) that its (yet unknown) spin is 1/2 and applied his approach in a joint work with Raymond Stora, [**7**], that anticipated the discovery of the τ*-meson*. Two subsequent publications [8, 9] are concerned with applications of parity conservation to decay and annihilation processes marking his continued interest in discrete symmetries. André Martin recalls [CC13] that Michel has pointed out in a 1952 lecture (four years before the famous paper by Lee and Yang) that existing experiments do not prove parity conservation in the weak interaction. A systematic exposition of his early work is given in the 80-page long thesis of Louis Michel [13] that also displays the general universal Fermi interaction. In line with French tradition Michel had to defend a "second thesis" which consisted in discussing orally a problem proposed by the faculty a few days earlier. The subject was *selection rules for reactions between particles*, a topic studied by the young man earlier that year, [**12**]. The result is important: the author introduces a new quantum number, the *isotopic parity*, a concept made popular three years later by Lee and Yang who gave to it, unfortunately, the less expressive name of *G-parity*. His paper [10] on the μ-decay finds a continuation in a joint work with Arthur Wightman [**15**] during Michel's first visit to Princeton. The early stage of his study of polarization of relativistic electrons was completed in a couple of papers [**19**, 20] with Claude Bouchiat (the second, more detailed one, being in French).

A visit of Michel to the Princeton University in 1958 gave rise to a new collaboration and to the celebrated work of Bargmann, Michel, Telegdi [**23**], that introduced a simple covariant equation for the precession of polarization of a relativistic charged particle. In fact, Michel has lectured on the background of this subject in Varenna in 1958 before the "BMT paper" was written - see [22] where the (pseudo)vector $W_\mu = 1/2\varepsilon_{\mu\nu\rho\sigma}P^\nu M^{\rho\sigma}$ (nowadays called the Pauli-Lubanski vector) which plays a central role in [**23**] is also used for the covariant description of spin.

In an email to Thibault Damour of January 17, 2001 Valentine Telegdi (1922-2006) explains the great popularity of this paper (which has by now some 1000 citations) by the fact that it was his (Telegdi's) simpler version of the article that was published[3]. He blames the "hermetic" (high-brow mathematical) "language of which Louis was an apostle" for the fact that Michel has not been credited for a number of his discoveries; he offers a parallel with the great Swiss theoretical physicist Stueckelberg who did not make easy the task of his readers either. At the origin of the attitude criticized by Telegdi was, in fact, Louis Michel's uncompromising integrity: he believed that when talking mathematics one should speak the language of mathematicians; in order not to confuse physical reasoning with a rigorous argument he even used two different blackboards: one for physics and another for mathematics.

Michel's interest in the topic of polarization continues through his joint work with François Lurçat [28, 29] on the relations between charge and spin (see also

[3]There exists indeed an earlier typewritten version that has twice as many formulas.

[30]). It is resumed at a more advanced level over ten years later in a series of papers that include his Catalan and French students Manuel Doncel and Pierre Minnaert, [53, 55, 56, 57, 58, 64, 60, 62, 68, 66]. All this work remains, regretfully, outside the scope of the present volume.

2. Group extensions. Internal symmetries and relativistic invariance [31, 34, 43]

As already noted, it was characteristic for Louis Michel when encountering a mathematical problem in physics not to be content with borrowing a ready formula but to master the background principles. A systematic study of symmetries under reflections (including charge conjugation and time reversal) and more generally, combining internal and (relativistic) space-time symmetries requires the study of group extensions. The superficial use of the notions of direct and semidirect products of groups (familiar to interested physicists) was not good enough.

Michel's entrance into the subject is documented (at least partly) in his papers at IHES. Having lectured on group theory in 1958 in Varenna he starts working on an ambitious project testified by a unpublished (23-typewritten-page) manuscript *Extensions du groupe de Lorentz par un groupe de jauge*. The authors feel that they have entered an unexplored territory and turn for advice to Serre[4]. Serre seems to be intrigued and puzzled: he starts his 3-page long answer by acknowledging that the concepts his physics colleagues are studying are not familiar to mathematicians: "... j'ignore comment on pourrait definir $H^q(B, A)$ lorsque B et A sont tous deux des groupes topologiques, ce qui est exactement le cas dont vous avez besoin ...". The outcome is that Michel in his 1962 lectures will restrict himself to the case where B is a discrete group - a case studied earlier by mathematicians. Let us note that Serre gives a course *Cohomologie Galoisienne* at Collège de France also in the academic year 1962-63 (published as a book in 1964). We leave it to historians of science to explore the influence of the above exchange to the subsequent work of the participants.

The need for a thorough study of the subject and the readiness to absorb the associated more abstract notions of group cohomology is testified in the early sixties by the number of authoritative schools and conferences devoted to this topic. We have selected here just the first of Michel's lecture courses (and review articles) devoted to the subject: besides **[31]** and its more advanced French version [32] here belong his already mentioned earlier work with Lurçat [28, 29] as well as the lecture notes [33, 36, 37, 40] among others. The interested reader is invited to also consult the subsequent courses which contain additional material (and do not repeat everything said in the first lecture series).

Paper **[34]** on the relation between internal symmetry and relativistic invariance anticipates and explains in clear terms the negative results of the subsequent work [35]. His lecture **[43]** reflects Michel's mature view on the status of the CP, T and CPT invariance. From a theoretical point of view this article is up-to-date and can be profitably used, for instance, in modern studies of CP violation in B-decays (or

[4] Jean-Pierre Serre (b. 1926 - three years junior to Michel) is by then a Fields medalist (since 1954 - the all time youngest) and Professor at Collège de France.

in theoretical discussions of the strong CP problem). For a related development in another domain of physics with the participation of the author - see Example 7.2 in [139] pp. 66-70.

3. A geometric view on broken symmetries [42, 80, 72, 86]

Michel strove to understand the essence of each problem, to strip it from accidental conventions and unnecessary assumptions. It was therefore natural for him to look for the geometric meaning of symmetry breaking. To use his words of **[86]** *it is comforting to the author of a physical model to predict* the critical orbits describing a spontaneously broken symmetry *but it is wise to remember that this is not a specific prediction of the model: it is simply the verification of a general geometric theorem.* The favoured (dynamically broken) flavour $SU(3)$ symmetry in the 1960's was the Gell-Mann and Ne'eman [G-MN64] "eightfold way". Not surprisingly Michel, together with his friend Luiggi Radicati, looked into the geometry of the octet representation of the unitary group $SU(3)$ in order to understand the underlying principles of the model that was in vogue at that time [**42**, 50, 59]. Michel's lecture [**42**] at the 1968 Coral Gables conference contains the basic ideas. The novelty of the authors' approach is illustrated by the subsequent rather voluminous discussion. Leading physicists in the field fail to follow even the definition of one of the basic notions. The authors define a *charge* as a generator of the fundamental (3×3 matrix) representation of the Lie algebra $su(3)$ with two equal eigenvalues and indicate (in note 6) that they give rise (by commutation) to an operator in the (8-dimensional) adjoint representation with just three eigenvalues - proportional to $1, 0, -1$. Michel reiterates this property answering a question of the chairman, Edward Teller; Yuval Ne'eman counters that the charge of the quarks $(2/3, -1/3)$ would not fit - forgetting that the same generator that has an eigenvalue $2/3$ and two eigenvalues $-1/3$ in the (3-dimensional) fundamental representation has eigenvalues $1, 0, -1$ (with degeneracies) in the 8-dimensional (adjoint) one.

The subsequent papers [50, 59] extend the idea to direct products like $SU(3) \times SU(3)$ and to $SU(n)$, respectively. We note that starting with [50] the term *layer* of **[42]** (for the set of orbits with a given stabilizer) is replaced by *stratum*. The 1977 work **[72]**, an early representative of a long term collaboration - and friendship - with Lochlainn O'Raifeartaigh and Kamesh Wali, extends the outlook of the study to include gauge theories with monopole solutions and Higgs fields. In an important later work [106] a method of searching absolute minima of the potential (in an $SO(10)$ grand unified theory) was developed which is still being applied.

Michel is not satisfied with polishing the mathematics of a single physical example. The next paper selected for this volume, **[80]**, is concerned with bifurcation phenomena in a rotating self-gravitating fluid. The reader is now invited to master a new field of physics and another chapter of group theory. The last paper of this selection, **[86]**, gives a nice overview of the field.

4. Nonlinear group action. Critical orbits [51, 52, 98, 76]

Thinking about symmetry breaking in particle physics Michel was confronted with the general notion of a group action. He soon realized that the mathematical texts addressed to physicists did not treat the problem in its natural generality. In the words of his 1980's talk [**86**]: "Fifty years ago were published the fundamental books of Weyl and of Wigner on application of group theory to quantum mechanics; since, some knowledge of the theory of linear group representations has become necessary to nearly all physicists. However the most basic concepts concerning group actions are not introduced in these famous books and, in general, in the physics literature."

Moreover, even in the specialized mathematical literature one is usually concerned with the physically uninteresting case of a homogeneous space of orbits. So Michel felt the necessity to study the notion of a critical orbit in a general, purely mathematical context [**51**] - while continuing at the same time (in the early 1970's) his work with Radicati on the applications of this notion to symmetry breaking phenomena in particle physics. It is ironic that the notion of a critical orbit of a group action in the context of a variational principle is usually attributed (in the mathematical physics literature) to the Palais article [P79] (which complains that Coleman's 1975 Erice lecture [C85] involves "unstated hypotheses" but does not mention the earlier work of Michel where all relevant assumptions are spelled out). The interested reader will recognize that the "reasonably broad" conditions (compact group action on a Riemannian manifold) for the validity of the "Principle of Symmetric Criticality" of [P79] are already contained in [**51**] and applied to real physical problems in the parallel work [50] and especially in Michel's lecture's [**52**]. The notions of group action, strata, space of orbits, critical orbits, ... provide a natural language in the study of symmetry and its breaking as illustrated in Michel's subsequent own work related to quite different domains of physics (see [**123**, 130, 131, 139, 140, 142]). Generically nonlinear group action induces, on the other hand, a linear representation on the space of invariant functions. Such a linearization is performed in [142] where the algebra of invariant functions on a Brillouin zone of two-dimensional crystals is studied through the explicit mapping to a higher dimensional space of invariant polynomials over $\sin\phi_i, \cos\phi_i$ new variables.

The more technical paper [**98**] illustrates the application of the algebra of invariants and covariants of finite groups. It has been noticed (over 50 citations) by both, applied physicists and symbolic computer mathematicians.

The talk [**76**] at the 1977 "Group theory" conference displays natural applications of generating functions of invariants and of the relation of symmetry with topology through Morse theory to solid state and molecular physics problems. Further applications of invariant theory to molecular and crystallographic problems can be found in [139, 140, 142] where the geometry of orbifolds is also exploited (see also the papers on periodic and aperiodic crystals of Sect. 6).

We also recommend to the interested readers the review article [83] which displays different physical applications of the same group action philosophy.

Singling out mathematical problems of group theory also accompanies later work of Michel (see [114, 116, 119, 121, 128, 133, 134]). The mere list of subjects: study of subgroups and subalgebras of Lie groups and algebras [114, 133], of Voronoi's cell of the weight lattice of a Lie algebra [134], symmetry classification of differential equations [116, 119], clearly shows that all these mathematical generalizations are based on some initial physical background and appear as mathematical questions inspired by physical problems.

5. Symmetry of matter. Defects. Phase transitions [71, 84, 97]. Having learned the basic theory of symmetry groups (stimulated by problems of particle physics), Louis Michel, in his late forties, is widening his scope both in the mathematical tools - incorporating discrete crystallographic groups and homotopy theory - and in the domain of applications, including the study of different phases of matter, problems of molecular and solid state physics. This turn is first displayed in a paper of 1972 (with Daniel Kastler et al.) [54] written in the elegant language of algebraic quantum field theory that appears uncommonly abstract (and hence difficult) to many of the potential users of the results. The symmetry groups of transitive Euclidean states are shown to split into five families of symmetry classes describing different forms of the matter organization. The result includes not only periodic crystals but also different kinds of liquid crystals (soft matter), macroscopic phases with broken symmetry, and even non-periodic (ergodic) states.

This approach was extended by Michel in the late 1970's, when he started analyzing topologically stable defects. The applications of the notion of homotopy to condensed matter physics, especially for the description of defects in soft matter, was initiated in the mid seventies by several independent groups. Michel wrote his first paper on this subject together with M. Kleman and G. Toulouse **[71]** in 1977. As usual, he was approaching the physical problem armed with the available mathematical tools of homotopy theory and trying to induce physicists to their use.

At the 1977 conference on Group Theoretical Methods in Physics in Tübingen, Michel lectured on "Topological classification of symmetry defects in ordered media". At that time, the list of publications on the subject was rather short (just some ten items are cited in [73]). In Michel's words physicists in fact used homotopy theory but like M. Jourdain of Molière's comedy "did not know that they are speaking prose". Michel did not want to follow such a physical style of discussion. Even in his short tourist guide on homotopy, appearing in [75], he uses such mathematical notions as long sequences of homotopy groups, thus introducing from the beginning the relevant mathematical language.

Michel's 1980 review article on symmetry defects and broken symmetry **[84]** (cited over 370 times) is a real gem. Starting with elementary, almost kindergarten example of a natural breaking of the symmetry of a square, passing through the Jacobi's rotating self-gravitating ellipsoid, the author leads the reader, gradually and with persuasion, to homotopy groups and the modern mathematical tools of category theory, exact sequences, commutative diagrams ... , explaining on the way defect formation and their stability, also reproducing the results of [54].

Passing from a pure symmetry analysis to a topological characterization is a very important step in our understanding of the structure of matter and of generic physical phenomena. In his subsequent work on applications of the study of group action to real crystals and to more general N-dimensional periodic structures, Louis Michel naturally tried to relate topology with symmetry using Morse theory [123], critical orbits, perfect Morse functions [139, 140], and studying global topological properties (connectivity [136] and monodromy [143]) of bands in periodic systems.

This line of thought is still very much alive. Recently, much work has been devoted to the description of different topological phases of matter. In particular, the classification of topological states of free electrons [K09] generalizes the characterization of the integer quantum Hall states by the topological "TKNN invariant" of [TKNN82] in physical terms, or by the first Chern class (in mathematics). The same kind of topological characterization can be applied to quantum systems of a finite number of particles via the concept of energy bands and associated topological invariants [FZ00].

On the other hand, the classification of topologically stable defects, applied in [84] to liquid crystals (and well known in the case of regular crystals) has found a natural application to the description of singularities of integrable dynamical systems associated with the notion of quantum monodromy (CD88). Generalization of defects and modification of their topology, inspired by physical examples, has in turn suggested the new mathematical concept of *fractional monodromy* (NSZ06), which can be considered as an example of the much more general mathematical construction of "wall crossing" [KS08].

In the 1980's Michel turned his attention to the symmetry analysis of (second order) phase transitions, work related to both the symmetry classification of different phases of matter discussed above and the symmetry breaking (considered in Sect. 3). This work [87, 96, 97, 101, 102, 103] (in which E. Brezin and J.C. and P. Toledano have also taken part) is based primarily on the observation that the phase transition occurs at fixed points of the (Wilson) renormalization group flow. Besides providing new illuminating proofs of known results the paper **[97]** establishes the uniqueness of a stable renormalization-group fixed points (whenever it exists). For further developments we refer to the more comprehensive (and better known) paper [103]. The study of renormalization group fixed points and phase transitions is still a hot topic (as the reader can easily convince himself looking through the electronic archive); for recent lecture notes on the subject including a bibliography - see [S12]. The contributions of Louis Michel to it, clear and to the point, are still valuable.

6. Periodic and aperiodic crystals [117, 123, 135].

Having worked on the general symmetry classification of different phases of matter, including defects in liquid crystals, Michel turned to a more specialized study of crystals. This theme first appeared in his publications within the discussion of symmetry breaking phenomena and phase transitions. The initial accent was put on the description of the universal behavior near the phase transition dealing with the Hamiltonians written as a truncated power series expansion [86, 87, 96].

Michel formulated a more general program to analyze crystal symmetry in his talk [90] at the 1981 International Conference on Mathematical Physics in Berlin (West). Pointing out that the description of symmetry in condensed matter is one of the oldest problem in mathematical physics, he notes that although the formulation of the most important initial crystallographic results obtained at the end of 19-th century, namely the existence of 230 crystallographic groups in three-dimensional space, is due to a close collaboration between the mathematician Schönflies and the mineralogist Fedorov, "it is difficult for many mathematical physicists to study crystallography because most of its fundamental concepts were conceived when group theory was less developed and they are defined only implicitly in the literature."

True to his scientific style, Michel first turned to the mathematical foundations of crystallography. He started with the description of symmetry structures of real crystals and their more abstract generalization: regular lattices in an N-dimensional space. An additional motivation for such a study came from the discovery at that time of a new class of substances forming the so-called *quasicrystals* or *aperiodic crystals*, as Louis Michel preferred to name them. These real three-dimensional solids can be described as projections to three-dimension of regular periodic structures in higher dimensional spaces. Michel actively participated in the discussions on this hot subject [108, 107, 110]. Not surprisingly, he based the systematic classification of arising structures in N-dimensional crystallography again on the notion of group action.

This did not look promising for practical applications because of the existence of the "International Tables of Crystallography", a Bible for crystallographers which collects the nomenclature, conventions and a detailed description of all groups for three- and two-dimensional spaces and which is largely used and referenced by any crystallographer. In spite of this, Michel insisted on his project in order to make the presentation of crystal structures and, in particular, the discussion of lattices as a special class of such structures, accessible to mathematically oriented readers.

The 1989 paper **[117]** (with Jan Mozrzymas) defines the basic concepts of crystallography in the language of group action. Michel's subsequent publications follow the same style of presentation and insist on the original definitions in order to educate the young generation of scientists in a mathematically sound terminology.

Trying to find a simple explanation of how the persistence of a long range order in crystals can follow from local interactions Michel returned to the work of B.N. Delone on "Delone sets" - a large family of point sets which includes the lattices [126]. According to his vision, in order to understand the physical periodic structures we need to start with general Delone sets and to find what local conditions are important in order to have global periodic structures. This kind of presentation of crystallography is partially realized by Michel in one of his last papers devoted to the study of crystal symmetry [141].

Description of lattices is naturally related to other important mathematical subjects: quadratic forms, elementary polyhedra and their combinatorial classification. Michel started, together with Marjorie Senechal and Peter Engel, a new big project : "Lattice Geometry" [146] which was supposed to include different mathematical approaches to the classification and description of lattices in arbitrary dimensions starting from initial basic definitions and going to the most advanced known results about N-dimensional lattices.

Another aspect of Michel's work related to the description of crystals is the classification of representations of space groups. He was not satisfied at all with the existing tables. His joint work with Bacry and Zak [111] on band representations is only available as a preprint but important partial results have appeared in [115, 120, 129, 136, 137, 143]. The accent in these publications falls on global topological properties of energy bands in solids.

In [**123**] the study of critical orbits on the Brillouin zone of three dimensional crystals is combined with Morse theory, thus giving the possibility to find the system of extremal points for a generic function on the Brillouin zone assuming that it has a minimal possible number of stationary points. The paper [**135**] treats physical situations in which crystal symmetry is extended by invariance under time reversal. The inclusion of additional discrete symmetries returns us on a new level to the early work of Michel on extended symmetry in particle physics. This paper is also interesting because it presents in a condensed form the system of polynomials forming a module of invariant functions that linearizes the strongly non-linear action of the symmetry group on the two-dimensional torus that represents the Brillouin zone of the crystal.

7. History of science. Scientific culture [XLIX, XXX, XL].

Having to select some three among the forty five articles of Louis Michel labeled as "of general or historical interest" in the bibliography has been particularly painful for us since they all give glimpses of Louis personality that we knew and loved.

The paper [**XLIX**] of 1999 provides a brief history of the Institut de Hautes Etudes Scientifique written on the occasion of the 40-th anniversary of the Institute. Louis Michel was the first physicist appointed as permanent professor at IHES and he has worked there nearly all these 40 years. The fact that he does not say much about himself in this article is also a characteristic of the man.

The talk [**XXX**] gives a personal view on a crucial period in the development of particle physics: the 1950's (as recalled some thirty years later). It helps understanding the evolution of scientific concepts and recognizing the contributions of different groups of physicists from a historical perspective.

Finally, the paper [**XL**] with the laconic title *Scientific culture* gives an opportunity to appreciate the scope of Louis Michel's interests spreading well beyond traditional science.

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